Reg. No. :

Question Paper Code : 97161

B.E./B.Tech. DEGREE EXAMINATION, DECEMBER 2015/JANUARY 2016.

Second Semester

Civil Engineering

MA 2161/080030004 — MATHEMATICS – II

(Common to All Branches)

(Regulations 2008)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the P.I of $y'' + y' = x^2 + 2x + 4$.

2. Convert the equation $(2x+3)^2 y'' - 2(2x+3)y' - 12y = 6x$ as a linear equation with constant co-efficients.

3. The temperature of points in space is given by $T(x,y,z) = x^2 + y^2 - z$. A mosquito located at $(1,1,2)$ desires to fly in such a direction that it will get warm as soon as possible. In what direction should it move?

4. State Green's theorem in a plane.

5. Is the function $f(z) = |z|^2$ analytic? Justify.

6. Define conformal mapping.

7. What is meant by essential singularity? Give an example.

8. State Cauchy's residue theorem.

9. Write down the existence conditions for Laplace transform. Is it necessary? Why?

10. Find the Laplace transform of $e^{-t}\int_0^t \frac{\sin t}{t}\, dt$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve : $y'' - 2y' + y = xe^x \sin x$. (8)

(ii) Using the method of variation of parameters, solve :

$$\frac{d^2y}{dx^2} + 4y = \tan 2x.$$ (8)

Or

(b) (i) Solve : $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$. (8)

(ii) Solve simultaneous differential equations

$$\frac{dx}{dt} + 2y = -\sin t \, ; \frac{dy}{dt} - 2x = \cos t$$ (8)

12. (a) (i) Show that $\overline{F} = \left(y^2 + 2xz^2\right)\overline{i} + (2xy - z)\overline{j} + (2x^2z - y + 2z)\overline{k}$ is irrotational and hence find its scalar potential. (8)

(ii) Using Stoke's theorem to evaluate $\int_C \overline{F} \cdot d\overline{r}$, where $\overline{F} = (\sin x - y)\overline{i} - \cos x\overline{j}$ and C is the boundary of the triangle whose vertices are $(0,0)$, $\left(\frac{\pi}{2}, 0\right)$ and $\left(\frac{\pi}{2}, 1\right)$. (8)

Or

(b) (i) Prove $\nabla^2(r^n) = n(n+1)r^{n-2}$ and deduce that $\frac{1}{r}$ satisfies Laplace equation. (6)

(ii) Verify Gauss divergence theorem for $\overline{F} = x^2\overline{i} + y^2\overline{j} + z^2\overline{k}$, where S is the surface of the cuboid formed by the planes $x = 0$, $x = a$, $y = 0$, $y = b, z = 0$ and $z = c$. (10)

13. (a) (i) State and prove the necessary conditions for $f(z)$ to be analytic. (8)

(ii) Find the bilinear transformation that maps the points $0, 1, \infty$ of the z-plane into the points $-5, -1, 3$ of the w-plane. Also find its fixed points. (8)

Or

(b) (i) Construct an analytic function $f(z) = u + iv$, given that $u = e^{x^2-y^2} \cos 2xy$. Hence find v. (8)

(ii) Discuss the transformation $w = \frac{1}{z}$. (8)

14. (a) (i) Use Cauchy's integral formula to evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-2)(z-3)} dz$, where C is the circle $|z| = 4$. (8)

(ii) Find the Laurent's expansion of $f(z) = \frac{7z-2}{(z+1)z(z-2)}$ in the region $1 < |z+1| < 3$. (8)

Or

(b) (i) Using Cauchy's residue theorem, to evaluate $\int_C \frac{z-3}{z^2+2z+5} dz$. where C is the circle $|z| = 3$. (8)

(ii) Evaluate $\int_0^{2\pi} \frac{\sin^2 \theta}{5 - 3\cos\theta} d\theta$, by using contour integration technique. (8)

15. (a) (i) Find the Laplace transform of : (1) $f(t) = t^2 \sin at$ (2) $f(t) = \frac{\cos at - \cos bt}{t}$. (4 + 4)

(ii) Solve $\frac{d^2x}{dt^2} + 9x = \cos 2t$, if $x(0) = 1, x\left(\frac{\pi}{2}\right) = -1$, using Laplace transform technique. (8)

Or

(b) (i) Find the Laplace transform of the function (8)

$$f(t) = \begin{cases} \sin wt, & 0 < t < \frac{\pi}{w} \\ 0, & \frac{\pi}{w} < t < \frac{2\pi}{w} \end{cases} \text{ and } f\left(t + \frac{2\pi}{w}\right) = f(t) \forall t.$$

(ii) Using convolution theorem, find the inverse Laplace transform of $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$ (8)